Brinson, Hood, and Beebower’s 1986 landmark study (henceforth, BHB) is well known to the portfolio management community. Yet, disagreements over BHB’s findings on asset allocation and its effect on a portfolio’s return, doubts about its relevance to investors, and varied interpretations of the research within the investment management industry have ignited a 25-year debate. To provide a framework for Vanguard’s own updated analysis and results, we first briefly review two studies at the core of this debate: BHB’s paper, “Determinants of Performance Portfolio” (1986), and William W. Jahnke’s rebuttal, “The Asset Allocation Hoax” (1997). We then expand upon Vanguard’s past research, most notably The Asset Allocation Debate: Provocative Questions, Enduring Realities by Davis, Kinniry, and Sheay (2007).

A look back at the asset allocation debate

Brinson and his colleagues concluded that a portfolio’s target asset allocation explained the majority of a broadly diversified portfolio’s return variability over time—that is, the extent to which actual returns diverged from returns of a long-term (and therefore static) asset allocation benchmark, also known as an investment policy allocation. These findings were subsequently confirmed by Ibbotson and Kaplan in 2000, as well as by Vanguard research (in Davis et al., 2007), suggesting that a portfolio’s investment policy is an important contributor to return variability (Hood, 2005).

In 1997, Jahnke argued that a focus on explaining return variability over time ignored the wide dispersion of total returns among broadly diversified active balanced funds at specific points in time. In other words, Jahnke claimed that a portfolio could achieve very different wealth levels as of the end of an investment time horizon, depending on which active fund or funds were selected. Jahnke’s analysis for Vanguard’s own updated analysis and results, we first briefly review two studies at the core of this debate: BHB’s paper, “Determinants of Performance Portfolio” (1986), and William W. Jahnke’s rebuttal, “The Asset Allocation Hoax” (1997). We then expand upon Vanguard’s past research, most notably The Asset Allocation Debate: Provocative Questions, Enduring Realities by Davis, Kinniry, and Sheay (2007).
emphasized that, as a result of active management strategies, actual returns should be compared across different active balanced funds with a set time horizon. (BHB’s intention was not to show that two funds using different strategies but the same asset allocation can have very different total returns over a specific time frame; it was to determine the extent to which asset allocation influenced return variability over time.)

As a result of our overall study, we suggest the following sequence for decision-making in the portfolio-construction process:

- The asset allocation policy decision should be the priority.
- The strategy used to implement it should be secondary.

**Our analytical framework**

To determine the relative performance of asset allocation and active management, we needed to distinguish between a portfolio’s policy return (or asset-allocation return)—that is, what a portfolio could have earned if it recreated its policy allocation with passively managed index funds—and its actual return, or what an active balanced fund earned over the period. Our empirical case tested BHB’s and Jahnke’s studies on a global scale, using a greater number of balanced mutual funds for the first time.

For our analysis, we selected balanced mutual funds from the Morningstar Direct database. The data included monthly net returns and fund characteristics, such as expense ratios and turnover rates. To ensure reliability, we only analyzed funds with at least 48 months of return history. We constructed each balanced fund’s policy portfolio using Sharpe-style returns-based analysis (1991). Among these funds, we selected total-return funds, income funds, asset allocation funds, and traditional balanced funds. For more details on our data and procedures, see the appendix. The box on page 9 lists benchmarks used in our analysis, by country.

**Conclusions from revisiting BHB**

As noted earlier, return variability measures the extent to which actual returns diverge from policy returns. Therefore, greater variability in returns would suggest a wider possibility of returns and a lessened ability to predict results, inherently indicating increased portfolio volatility. The variation in the policy return that explains the percentage of variation in the actual return is measured by R-squared ($R^2$), a gauge of how much of a portfolio’s performance can be explained by the returns from an overall market benchmark index.²

Because $R^2$ is derived from a time-series regression analysis of the fund’s actual return versus its policy return, a high $R^2$ would mean that variations in the policy return explained a high percentage of the variation in fund returns. (Regression analysis seeks to explain the nature and strength of the relationship between a dependent variable and one or more independent variables that determine the value of the dependent variable.)

BHB’s conclusions were derived from the results of a time-series analysis, which uses monthly return data over time for one “subject”—a single mutual fund, in this case. The goal was to measure the effect of asset allocation on return variability. Our results confirmed BHB’s findings that, on average, most of a portfolio’s return variability over time was attributed to the ups and downs of its long-term policy asset allocation. Active investment decisions—such as market-timing and security selection—had relatively little impact.

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¹ *Returns-based style analysis* is a statistical method for inferring a fund’s effective asset mix by comparing the fund’s returns with the returns of asset-class benchmarks. Developed by William F. Sharpe, RBSA is a popular attribution technique because it doesn’t require tabulating the actual asset allocation of each fund for each month over time; rather, it regresses the fund’s returns against the returns of asset-class benchmarks.

² All the $R^2$s in this article are “adjusted $R^2$s.” This modification of $R^2$ takes into account the differing number of data points in each fund’s return series to make the $R^2$s comparable.
It is important to note that BHB’s dataset consisted of pension funds, which are typically exposed to a high level of systematic market risk, that is, vulnerability to events that affect broad-market returns. This resulted in high $R^2$ numbers. BHB concluded that more than 90% of return variability over time could be explained by the asset allocation policy. As noted earlier, Ibbotson and Kaplan (2000) and Vanguard research in Davis et al. (2007) found similar results.

This concurred with our current research for the balanced mutual fund universes in the United States, Canada, the U.K., and Australia, although the percentages were slightly lower than BHB’s findings. As Figure 1 shows, for selected periods from January 1962 through December 2011, the median percentage of actual-return variation that was due to asset allocation ranged from 80% in the United Kingdom to 91.4% in the United States.

This reinforces our view that asset allocation should take priority over implementing a strategy—that asset allocation is key in managing the range of returns a portfolio may experience over time.

Conclusions from revisiting Jahnke

Jahnke (1997) focused on determining how much asset allocation affects actual portfolio return dispersion across funds, that is, the difference between multiple funds’ cumulative returns over the same time horizons relative to their passive policy benchmark returns. An $R^2$ was derived from a cross-sectional regression analysis, which uses data for multiple “subjects”—a number of actively managed mutual funds in this case—at one point in time.

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**Figure 1.** Role of asset allocation policy in return variation of balanced funds

*Selected periods, January 1962 through December 2011*

<table>
<thead>
<tr>
<th>United States</th>
<th>Canada</th>
<th>U.K.</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>518</td>
<td>245</td>
<td>294</td>
<td>336</td>
</tr>
</tbody>
</table>

Notes: For each fund in our sample, a calculated adjusted $R^2$ represented the percentage of actual-return variation explained by policy-return variation. Percentages stated in the figure—91.4%, 88.3%, 80.0%, and 89.9%, for the United States, Canada, the U.K., and Australia, respectively—represent the median observation from the distribution of percentage of return variation explained by asset allocation for balanced funds. For the United States, the sample included 518 balanced funds for the period January 1962–December 2011; for Canada, 245 balanced funds for January 1990–December 2011; for the U.K., 294 balanced funds for January 1990–December 2011; and for Australia, 336 balanced funds for January 1990–December 2011. Calculations were based on monthly net returns, and policy allocations were derived from a fund’s actual performance compared to a benchmark using returns-based style analysis (as developed by William F. Sharpe) on a 36-month rolling basis. Funds were selected from Morningstar’s Multi-Sector Balanced category. Only funds with at least 48 months of return history were considered in the analysis, and each fund had to have a greater-than-20% long-run equity exposure, both domestic and international (based on the average of all the 36-month rolling periods), and a greater-than-20% bond allocation (domestic and international) over its lifetime. The policy portfolio was assumed to have a U.S. expense ratio of 1.5 basis points per month (18 bps annually, or 0.18%) and a non-U.S. expense ratio of 2.0 bps per month (24 bps annually, or 0.24%).

Sources: Vanguard calculations, using data from Morningstar, Inc.
Both our and Jahnke’s analysis produced low $R^2$ numbers (see Figure 2): The median dispersion that can be explained by asset allocation was 38.0% in the United States, 22.5% in Canada, 23.0% in the U.K., and 32.8% in Australia. In other words, active management implemented by taking idiosyncratic risks and differential exposure to systematic risk (such as factor or tactical overweights) can create significant return dispersion across active balanced funds, resulting in a low $R^2$, indicating that asset allocation explains a minority of return dispersion at one point in time.

Jahnke believed that, rather than return variability, or the volatility experienced over time (BHB’s focus), investors care about actual returns and the range of possible investment outcomes at the end of their time horizons. Jahnke’s analysis confirmed that some actively managed funds can outperform their policy portfolios.

Vanguard’s previous and latest research supports BHB’s finding that broadly diversified balanced-fund returns move in tandem with broad markets over time. And Jahnke’s study found that actual returns can vary across funds over a specific time horizon.

Given the two different perspectives, investors’ essential question remains unresolved: Can active management increase a portfolio’s return without increasing the portfolio’s experienced volatility?

**What matters most to investors: Risk–return trade-off**

BHB’s most important contribution was to measure how much of a portfolio’s return variability came from these three components: asset allocation policy (based on indexes that did not change over time), security selection, and market-timing. They showed that, on average, the actively managed pension funds that they studied had been unable to add value through market-timing or security selection beyond their static indexed policy returns. This result was consistent with the observation that indexing outperforms a significant portion of active portfolios in equity and bond markets (Philips, 2012).

In our comparison of actual returns versus policy returns, we calculated the average return of a fund’s asset allocation policy as a percentage of the fund’s long-term average return. We also computed the ratio of a fund’s policy volatility over its actual volatility. These two calculations helped us determine how both an investor’s policy and active management strategies have performed in the past.
We found that actively managed funds added to volatility levels and underperformed the benchmark, on average. This is reflected in Figures 3 and 4, showing results for the United States, Canada, the U.K., and Australia.

In a separate analysis, our findings showed that a high percentage of actively managed balanced funds underperformed their policy portfolios, as follows:

- 68% of active balanced funds in the United States (January 1962–December 2011).
- 70% of active balanced funds in the U.K. (January 1990–December 2011).
- 77% of active balanced funds in Australia (January 1990–December 2011).
- 62% of active balanced funds in Canada (January 1990–December 2011).

Figure 3 measures the risk–return trade-off using the Sharpe ratio, which calculates the amount of return derived per unit of risk taken (by comparing equity-risk premiums and standard deviations). The higher the ratio, the better the investment’s risk-adjusted return.

Figure 3 shows a clear rise in Sharpe ratios for the policy returns over the fund’s actual returns. The higher risk taken in the fund relative to the policy comes from active management strategies such as market-timing and stock selection. (Figure 4 is discussed in more detail in the next section.)

We found that, on average, a greater degree of active management reduced both the time-series $R^2$ (as calculated in BHB’s study) and cross-sectional $R^2$ (as developed in Jahnke’s 1997 study), but did not necessarily increase performance. As Sharpe has noted, on average, active management risk is not compensated—unless skill overcomes hurdles such as tendencies toward higher costs and turnover of active management (Sharpe, 1991).

In addition, in recent Vanguard research, Wallick, Bhatia, and Cole (2010) attempted to quantify an optimal active and index allocation for investors who have differing skill levels for choosing managers who outperform their benchmark; that study’s results showed that indexing was valuable for all investors when considering the level of return per unit of risk taken.

**Characteristics of funds with positive and negative alpha**

In addition to showing that the average actively managed fund reduced returns and increased return variability compared with funds that mirrored the policy benchmark, our analysis found that some actively managed balanced funds have significantly outperformed their policy benchmarks over time. What are the general characteristics of these “winning” funds? And how do they compare with the broader universe of active balanced funds?
**Figure 4. Fund characteristics across study’s funds: United States, Canada, U.K., Australia**

<table>
<thead>
<tr>
<th></th>
<th>All balanced funds</th>
<th>Funds with statistically significant positive alpha</th>
<th>Funds with statistically significant negative alpha</th>
<th>Funds with zero alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S. balanced funds (January 1962–December 2011)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk and return (average across funds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annualized alpha</td>
<td>–0.76%</td>
<td>2.51%</td>
<td>–2.65%</td>
<td>–0.54%</td>
</tr>
<tr>
<td>Policy return as percentage of actual return</td>
<td>104.9%</td>
<td>72.7%</td>
<td>122.8%</td>
<td>102.9%</td>
</tr>
<tr>
<td>Policy volatility as percentage of actual volatility</td>
<td>93.9%</td>
<td>93.6%</td>
<td>96.7%</td>
<td>93.2%</td>
</tr>
<tr>
<td>Return variability explained by policy variability</td>
<td>87.9%</td>
<td>84.4%</td>
<td>91.2%</td>
<td>87.4%</td>
</tr>
<tr>
<td><strong>Average fund characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expense ratio</td>
<td>0.89%</td>
<td>0.70%</td>
<td>1.17%</td>
<td>0.84%</td>
</tr>
<tr>
<td>Net assets ($ millions)</td>
<td>$781.3</td>
<td>$5,231.4</td>
<td>$419.5</td>
<td>$552.2</td>
</tr>
<tr>
<td>Turnover</td>
<td>69.59%</td>
<td>67.13%</td>
<td>83.04%</td>
<td>66.62%</td>
</tr>
<tr>
<td><strong>Funds in sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of funds</td>
<td>518</td>
<td>28</td>
<td>93</td>
<td>397</td>
</tr>
<tr>
<td>Percentage of all funds</td>
<td>5%</td>
<td>18%</td>
<td>77%</td>
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<tr>
<td><strong>Canadian balanced funds (January 1990–December 2011)</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk and return (average across funds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annualized alpha</td>
<td>–0.29%</td>
<td>3.51%</td>
<td>–1.69%</td>
<td>–0.44%</td>
</tr>
<tr>
<td>Policy return as percentage of actual return</td>
<td>102.6%</td>
<td>77.4%</td>
<td>118.0%</td>
<td>102.0%</td>
</tr>
<tr>
<td>Policy volatility as percentage of actual volatility</td>
<td>93.0%</td>
<td>93.8%</td>
<td>96.7%</td>
<td>92.0%</td>
</tr>
<tr>
<td>Return variability explained by policy variability</td>
<td>82.0%</td>
<td>68.4%</td>
<td>91.5%</td>
<td>81.4%</td>
</tr>
<tr>
<td><strong>Average fund characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expense ratio</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Net assets (C$ millions)</td>
<td>C$367.6</td>
<td>C$679.4</td>
<td>C$414.3</td>
<td>C$315.2</td>
</tr>
<tr>
<td>Turnover</td>
<td>36.60%</td>
<td>25.17%</td>
<td>32.59%</td>
<td>39.10%</td>
</tr>
<tr>
<td><strong>Funds in sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of funds</td>
<td>245</td>
<td>23</td>
<td>45</td>
<td>177</td>
</tr>
<tr>
<td>Percentage of all funds</td>
<td>10%</td>
<td>18%</td>
<td>72%</td>
<td></td>
</tr>
<tr>
<td><strong>U.K. balanced funds (January 1990–December 2011)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk and return (average across funds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annualized alpha</td>
<td>–1.10%</td>
<td>5.05%</td>
<td>–3.81%</td>
<td>–1.04%</td>
</tr>
<tr>
<td>Policy return as percentage of actual return</td>
<td>105.4%</td>
<td>70.5%</td>
<td>131.8%</td>
<td>103.4%</td>
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</tr>
<tr>
<td>Return variability explained by policy variability</td>
<td>75.8%</td>
<td>54.9%</td>
<td>81.2%</td>
<td>76.1%</td>
</tr>
<tr>
<td><strong>Average fund characteristics</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expense ratio</td>
<td>1.49%</td>
<td>1.35%</td>
<td>1.65%</td>
<td>1.48%</td>
</tr>
<tr>
<td>Net assets (£ millions)</td>
<td>£98.8</td>
<td>£189.1</td>
<td>£81.6</td>
<td>£96.4</td>
</tr>
<tr>
<td>Turnover</td>
<td>67.05%</td>
<td>68.93%</td>
<td>69.00%</td>
<td>66.67%</td>
</tr>
<tr>
<td><strong>Funds in sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of funds</td>
<td>294</td>
<td>13</td>
<td>35</td>
<td>246</td>
</tr>
<tr>
<td>Percentage of all funds</td>
<td>4%</td>
<td>12%</td>
<td>84%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Funds with consistently positive (or negative) excess return (alpha) had statistically significant alpha using a 95% one-sided t-test for statistical significance. N.A. = not available (insufficient data to provide an accurate metric).

Sources: Vanguard calculations, using data from Morningstar, Inc.

(Continued on page 12)
Figure 4 (Continued). Fund characteristics across study’s funds: United States, Canada, U.K., Australia

<table>
<thead>
<tr>
<th>Australian balanced funds (January 1990–December 2011)</th>
<th>All balanced funds</th>
<th>Funds with statistically significant positive alpha</th>
<th>Funds with statistically significant negative alpha</th>
<th>Funds with zero alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk and return (average across funds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annualized alpha</td>
<td>–0.81%</td>
<td>1.34%</td>
<td>–1.83%</td>
<td>–0.51%</td>
</tr>
<tr>
<td>Policy return as percentage of actual return</td>
<td>105.9%</td>
<td>91.9%</td>
<td>113.9%</td>
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<td>99.0%</td>
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<td>92.1%</td>
</tr>
<tr>
<td>Return variability explained by policy variability</td>
<td>86.2%</td>
<td>94.6%</td>
<td>89.1%</td>
<td>84.8%</td>
</tr>
<tr>
<td>Average fund characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expense ratio</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Net assets (A$ millions)</td>
<td>A$59.2</td>
<td>A$196.3</td>
<td>A$16.0</td>
<td>A$70.3</td>
</tr>
<tr>
<td>Turnover</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Funds in sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of funds</td>
<td>336</td>
<td>8</td>
<td>87</td>
<td>241</td>
</tr>
<tr>
<td>Percentage of all funds</td>
<td>2%</td>
<td>26%</td>
<td>72%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Funds with consistently positive (or negative) excess return (alpha) had statistically significant alpha using a 95% one-sided t-test for statistical significance. N.A. = not available (insufficient data to provide an accurate metric).

Sources: Vanguard calculations, using data from Morningstar, Inc.

Figure 4 sorts our study sample’s 518 U.S. balanced funds into three cohorts, based on their average alpha, or a risk-adjusted measure of the “excess returns” (the difference between the funds’ returns and that of their estimated policy benchmarks).

- 5% of the sample, or 28 funds, posted a statistically significant positive excess return.
- 18% of the sample, or 93 funds, significantly trailed the performance of their policy allocations.
- 77% of the sample, or 397 funds, had an alpha of approximately zero.3

In the United States, the funds that consistently underperformed trailed their policy benchmarks by an average of –2.7 percentage points per year. As shown in the figure, outperforming funds achieved higher returns than their policy allocations (72.7% policy-to-actual-return ratio) by incurring more active management risk (93.6% policy-to-actual-volatility ratio). Conversely, underperforming funds earned a lower return than their policy allocations (122.8% policy-to-actual-return ratio) while incurring more active management risk than their benchmarks (96.7% policy-to-actual-volatility ratio). The results for Canada, the U.K., and Australia were generally similar.

Although manager skill certainly plays a role in distinguishing positive-alpha from negative-alpha funds, other differences shown in Figure 4 are noteworthy. In general, we found that “winning” active funds had lower expenses, lower portfolio turnover, and more assets under management than the consistently underperforming funds.

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3 Funds whose excess returns were statistically different from zero at the 95% confidence level using a one-side t-test were classified into the “statistically significant alpha” categories.
Conclusions for all markets

Results of Vanguard’s latest research for U.S., Canadian, U.K., and Australian funds were proportionately much the same in terms of the degree to which asset allocation was found to explain return variability versus benchmarks over time and dispersion of returns across funds. Our analysis—which expanded upon the work of BHB, whose findings were later confirmed in Vanguard research—reinforced the view that asset allocation explains the majority of a portfolio’s return variability. For investors who held broadly diversified portfolios, asset allocation was the primary driver for return variability. In addition, we found that indexed policy portfolios provided, on average, higher returns and lower volatility than actively managed funds.

Furthermore, we concluded that the portfolio construction process should begin with an investor choosing an asset allocation policy. An investor can then determine the strategy for implementing the policy decision, based on his or her risk-and-return expectations.

References

BHB. See Brinson, Hood, and Beebower (1986).


Benchmarks used in our analysis (all returns in local currency):


Appendix. Empirical methodology and fund characteristics for Canada, U.K., and Australia

1. Estimation of policy allocation

The policy weightings, or asset allocation, for each fund were estimated by performing returns-based style analysis over each fund’s rolling three-year history. Style analysis (Sharpe, 1988) is a statistical method for inferring a fund’s effective asset mix by comparing the fund’s returns with returns of asset-class benchmarks. Style analysis is a popular attribution technique because it does not require tabulating the actual asset allocation of each fund for each month over time. Rather, style analysis facilitates return attribution by regressing the return of the fund against the returns of asset-class benchmarks. The following regression was estimated:

\[ r_{t}^\text{fund} = \alpha + w_{t}^\text{stock} r_{t}^\text{stock} + w_{t}^\text{bond} r_{t}^\text{bond} + w_{t}^\text{cash} r_{t}^\text{cash} + \varepsilon_{t} \]

For our purposes, style analysis required not only that the asset-class weight parameters sum to 1, but also that each asset-class weight be positive (no short sales).

2. Calculation of policy return

\[ r_{t}^\text{policy} = w_{t}^\text{stock} r_{t}^\text{stock} + w_{t}^\text{bond} r_{t}^\text{bond} + w_{t}^\text{cash} r_{t}^\text{cash} - \text{cost} \]

Cost is the approximate expense ratio, as a percentage of assets, of replicating the policy mix using indexed mutual funds. The policy portfolio was assumed to have a U.S. expense ratio of 1.5 bps per month (18 bps annually, or 0.18%) and a non-U.S. expense ratio of 2.0 bps per month (24 bps annually, or 0.24%).

3. Time-series regression of actual returns against policy returns

To compare variation in the policy and actual returns, we calculated an R^2 for each fund by regressing its actual return against its policy return:

\[ r_{t}^\text{fund} = \alpha + \beta r_{t}^\text{policy} + \varepsilon_{t} \]

---

4 Additional asset-class benchmarks may be used for non-U.S. mutual fund markets, expanding the equation with the appropriate added terms.
4. Cross-sectional regression of actual returns against policy returns

To compare variation in the policy and actual returns across different funds, we calculated an $R^2$ in a given month by regressing the actual returns against the policy returns for all funds in that month:

$$r_{\text{fund}} = \alpha + \beta r_{\text{policy}} + \varepsilon_i$$

5. Ratio of the cumulative policy return to the cumulative actual return

The policy return as a percentage of the actual return of each fund is the ratio of its cumulative policy return to its cumulative actual policy return. When cumulative policy return is greater than cumulative actual return, this ratio is greater than 100%.

$$N \prod_{t=1}^N \frac{(1+r_{\text{policy}})}{(1+r_{\text{fund}})}$$

6. Ratio of policy volatility to actual volatility

The policy volatility as a percentage of the actual return volatility of each fund is the ratio of the standard deviation of the policy return to the standard deviation of the actual return. When policy return volatility is smaller than actual return volatility, this ratio is less than 100%.

$$\sqrt{\frac{1}{N-1} \sum_{t=1}^N [r_{\text{policy}}^t]^2 - \left(\frac{1}{N} \sum_{t=1}^N r_{\text{policy}}^t\right)^2}$$

$$\sqrt{\frac{1}{N-1} \sum_{t=1}^N [r_{\text{fund}}^t]^2 - \left(\frac{1}{N} \sum_{t=1}^N r_{\text{fund}}^t\right)^2}$$

### Formula components

- $w_s$ = policy allocation attributed to stocks, ranges from 0 to 1
- $w_b$ = policy allocation attributed to bonds, ranges from 0 to 1
- $w_c$ = policy allocation attributed to cash, ranges from 0 to 1
- $r_{\text{stock}}^t$ = return on the equity benchmark in period $t$
- $r_{\text{bond}}^t$ = return on the bond benchmark in period $t$
- $r_{\text{cash}}^t$ = return on the cash benchmark in period $t$
- $\alpha$ = excess return of the fund that cannot be attributed to benchmark returns
- $\varepsilon_{i,t}$ = residual that cannot be explained by the asset-class returns
- $r_{\text{fund}}^t$ = total return of the fund in period $t$
- $r_{\text{policy}}^t$ = total return of the policy in period $t$
- $r_{\text{fund}}^i$ = total return across funds
- $r_{\text{policy}}^i$ = total return across policies
- $\beta$ = sensitivity of changes in the fund return to changes in the policy return
- $N$ = total number of monthly net returns for each fund
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