Vanguard research

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# The Vanguard Asset Allocation Model: An investment solution for active-passive-factor portfolios

- Mean-variance optimization and other conventional portfolio construction approaches operate in two dimensions: portfolio risk and portfolio return. Real-world investor decisions, however, suggest that portfolio selection depends on the intersection of multiple dimensions of risk and return, from systematic risk and volatility to active alpha, tracking error, and implicit risk factor exposures.
- The Vanguard Asset Allocation Model (VAAM), a proprietary model for determining asset allocation among active, passive, and factor investment vehicles, simultaneously optimizes across three dimensions of risk-return trade-offs: alpha, systematic, and factor. The model incorporates Vanguard's forward-looking capital market return projections as well as client expectations for alpha risk and return to create portfolios consistent with the full set of investor preferences.
- The VAAM can solve portfolio construction problems that conventional approaches address in an ad hoc and suboptimal manner. The result is more appropriate active-passive-factor portfolio solutions to common investor objectives and asset allocation problems. These solutions include multiasset portfolios that help achieve investor objectives such as (1) wealth growth, (2) risk hedging, and (3) return targets using investment methodologies including model-based strategic asset allocation, active-passive asset allocation, and time-varying asset allocation.


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## Asset allocation and the need for an active-passive model

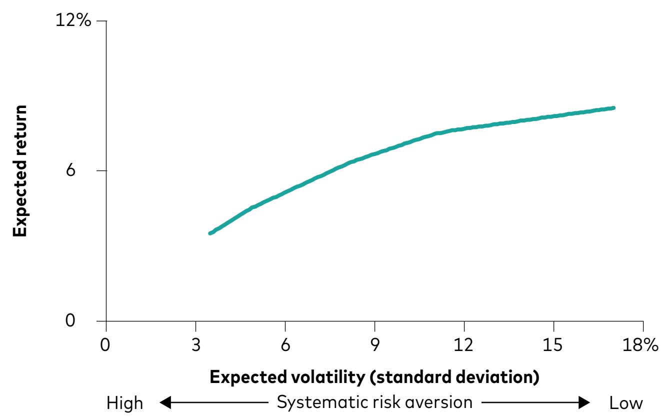
The debate between active and passive investing is well documented. Despite the underperformance of the average active manager net of costs (see Sharpe, 1991, and Rowley, Harbron, and Tufano, 2017), many investors still allocate to active management. This suggests that traditional portfolio construction may not fully address all investors' preferences and beliefs. After all, some active managers do outperform before costs, and investors with active allocations must believe with some conviction that they can select managers from the "right half" of the distribution.

In 2017, Vanguard introduced a framework designed to help investors make more informed decisions when allocating across active and passive investments (see Wallick et al., 2017, and Aliaga-Díaz et al., 2024). This framework considers alpha risk (or active risk) and an investor's tolerance for it, a component missing in traditional mean-variance optimization (MVO) approaches to active strategies and factor investing. Including active risk aversion transforms the two-dimensional MVO efficient frontier, as illustrated in **Figure 1a**, into a three-dimensional efficient frontier—an efficient surface—as illustrated in **Figure 1b**. (For more information about the construction of an efficient frontier, see page 9.)

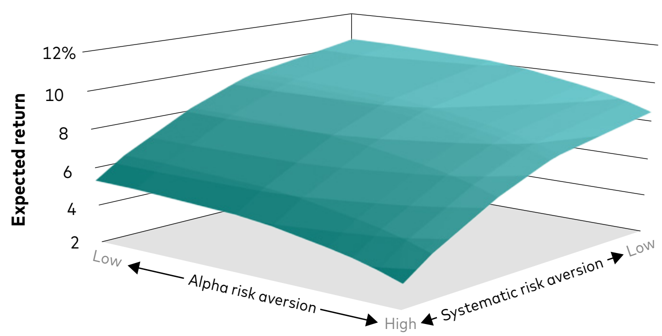
**FIGURE 1**

**The missing link: Alpha risk aversion**

a. Traditional MVO efficient frontier




b. Efficient frontier with alpha risk




**Note:** This graph is for illustrative purposes only and does not represent any particular investment.

**Source:** Vanguard.

The Vanguard Asset Allocation Model (VAAM) grew from the need to help investors address the investment trade-offs they face across layers of alpha, systematic (or passive), and factor risk. The VAAM is a utility-function-based model that integrates forward-looking return expectations generated by the Vanguard Capital Markets Model® (VCMM) to determine optimal portfolio allocations across active, passive, and factor investments. The VAAM uses inputs like an investor's risk tolerance, investment horizon, and asset-class preferences to assess risk-return trade-offs and generate a range of portfolio metrics such as forward-looking risk and return distributions, expected maximum drawdown, and probability of returns exceeding certain levels.

The VAAM was built on the principles of the Vanguard active-passive framework, but its application extends beyond this foundational use. It offers a comprehensive investment solution that addresses real-world portfolio challenges. It not only helps investors decide on the best mix of active and passive strategies across different asset classes, but it also considers the impact of each such decision on overall asset allocation. In addition, the VAAM explores how factor investing can be integrated with active management styles and how these decisions vary with investor risk tolerance and market conditions, providing tailored strategies for diverse scenarios.

This paper is organized into five sections. We begin with the foundational question that motivated the creation of the VAAM: What sources of traditional active fund returns are crucial for an active-passive allocation? We then provide an overview of the VAAM optimization framework, describing the model's key inputs such as asset-return expectations, portfolio constraints, and an investor's attitude toward various dimensions of risk. The third section combines the first two and illustrates the sensitivity of VAAM-customized portfolios to a full range of potential investor inputs. Next, we provide an overview of Vanguard's portfolio construction framework and the general application of the VAAM, followed by a few specific portfolio applications of the VAAM, such as advantages of optimized portfolios over ad hoc ones, active-passive portfolios versus market-cap-weighted ones, and return-target portfolios versus static ones. Finally, we discuss the VAAM's caveats and limitations.

**IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from the VCMM are derived from 10,000 simulations for each modeled asset class. Simulations are as of March 31, 2024. Results from the model may vary with each use and over time. For more information, please see "About the Vanguard Capital Markets Model" on page 20.**

## Motivations for the VAAM: Considering active strategy elements

There are three key attributes of any active strategy that need to be considered in the active-passive allocation problem. The VAAM allows us to obtain the optimal active allocation based on three key attributes:

1. **Factor-adjusted alpha**, or alpha that excludes performance gains from an active manager's factor tilts;
2. **Alpha risk**, or the chance that an active manager outperforms or underperforms their passive counterpart; and
3. **Alpha risk aversion**, or the degree to which investors seek to avoid alpha risk and possible underperformance in pursuit of outperformance.

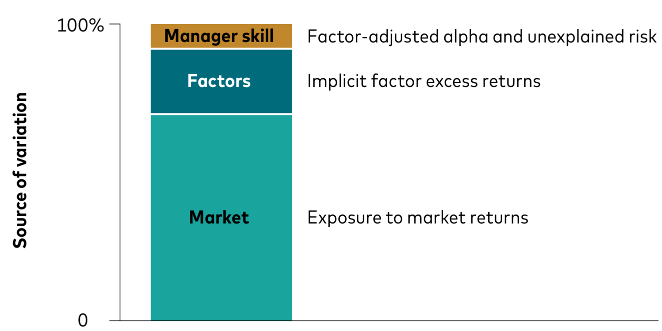
### Factor-adjusted alpha: The true measure of manager skill

Should active fund managers be credited for outperformance due to systematic factor tilts known to harvest long-term risk premia? Research indicates that much of the performance attributed to active managers in both equities and bonds can be replicated through factor exposure. For example, studies show that a significant portion of active U.S. equity managers' alpha comes from equity risk factors, and most returns from active fixed income managers stem from exposure to credit and high-yield securities rather than market-timing or security selection (see Roberts, Paradise, and Tidmore, 2018, and Bender, Hammond, and Mok, 2014). This suggests that true active management skill should focus on security selection and timing, especially considering that factor exposure can often be found at a lower cost than active management fees.

Following the logic of a risk-factor attribution least-squares regression (see Sharpe, 1992, Fama and French, 1993, and Chin and Gupta, 2017), **Figure 2** shows how active fund returns can be decomposed into a market component (or systematic risk), a risk factor component, and a manager skill component consisting of factor-adjusted alpha and unexplained return variation (or tracking error). This approach for estimating factor-adjusted alpha can be a valuable tool for investors in assessing active fund managers and how much value they add.[1]

**FIGURE 2**

**Factor decomposition of active returns**




**Note:** This graph is for illustrative purposes only and does not represent any particular investment.

**Source:** Vanguard.

1 In this paper, we focus on equity style factors only. For further details on how we define U.S. equity style factors for the purposes of this paper, please see Appendix A on page 22. However, the approach and methodology that we propose is applicable to any definition of factors and across different asset classes, including typical fixed income factors (such as duration and credit) and factor replication for alternative strategies (such as hedge funds).

**Figure 3** shows the return decomposition for a hypothetical U.S. equity active manager. The active fund shows strong factor-adjusted outperformance, with a net alpha of 83 basis points (bps) per year and a tracking error of roughly 4%. (A basis point is one-hundredth of a percentage point.) That is, the fund manager has added value through security selection and timing, beyond traditional factor and market exposure. In this example, the active fund shows exposure to the illiquidity factor and a value tilt.

FIGURE 3

### Factor decomposition of a U.S. equities active fund

| Category | Component | Value |
|---|---|---|
| **Factor-adjusted alpha** | Annualized alpha | 0.83%* |
| **Market beta and factor loadings** | Market beta | 0.99** |
| | Value | 0.05 |
| | Illiquidity | 0.05** |
| **Return-based regression statistics** | Adjusted R-squared | 96.7% |
| | Tracking error | 4.03% |
| | Information ratio | 0.30 |

* Indicates statistical significance at 5%;

** Indicates statistical significance at 1%.

**Notes:** The active fund is a hypothetical but realistic representation of an active manager's performance and factor tilts. The data are estimates based on the ordinary least-squares (OLS) regression of historical active manager returns; the value and illiquidity factors were selected randomly. For more details on how factors are constructed, see Appendix A on page 22. Using this specific hypothetical active fund highlights how the VAAM would react to an investor who is willing to have value and illiquidity factor exposure in their portfolio, either through implicit active manager exposure or through a passive factor investment.

**Source:** Vanguard.

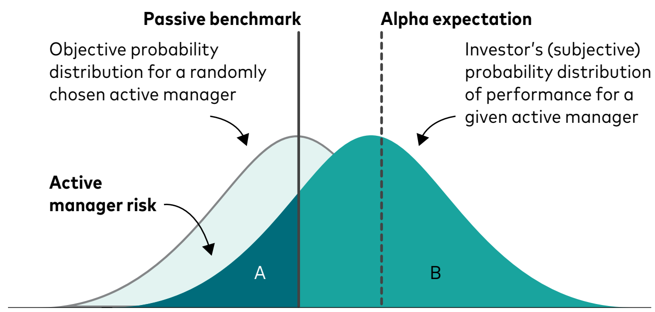
## Alpha risk: The uncertainty around factor-adjusted alpha

Investors in active funds generally expect outperformance compared to passive alternatives, holding a positive, though uncertain, expectation of factor-adjusted alpha. Even top-performing active managers can face periods of underperformance. This concept is depicted in **Figure 4**, where active manager performance is shown as a probability distribution. While there's a positive alpha expectation (shown as a dotted line), underperformance relative to passive benchmarks (labeled A) is possible.

Investors, many of whom tend to be risk-averse, weigh this potential underperformance against positive outcomes (labeled B). This distributional view of active manager skill, focusing on the statistical mean and standard deviation from performance outcomes, adds a nuanced perspective often overlooked in the traditional active-passive debate, which usually considers alpha as a fixed-point forecast.[2]

FIGURE 4

### Alpha expectation and active risk




**Notes:** This graph is for illustrative purposes only and does not represent any particular investment. The size of the area representing the probability of the active fund to underperform its passive counterpart (labeled A) is ultimately a function of the associated level of active risk (or tracking error).

**Source:** Vanguard.

## Alpha risk aversion: The investor's attitude toward alpha risk

Research from Aliaga-Díaz et al. (2024) explores how investors' appetite for alpha risk can influence their decisions when choosing between active and passive management. Just as investors vary in their aversion to systematic risk, they also differ in their aversion to alpha risk. This tolerance affects their investment choices, with those more averse to alpha risk typically allocating less to active strategies.

The concept of balancing expected alpha against alpha risk aligns with earlier studies by Flood and Ramachandran (2000), Waring et al. (2000), and Waring and Siegel (2003), which treat the active-passive decision as a risk-budgeting problem and provide quantitative methods for determining optimal active allocations.

**2** Notable exceptions to this fixed-point forecast approach include Flood and Ramachandran (2000), Waring et al. (2000), and Waring and Siegel (2003).

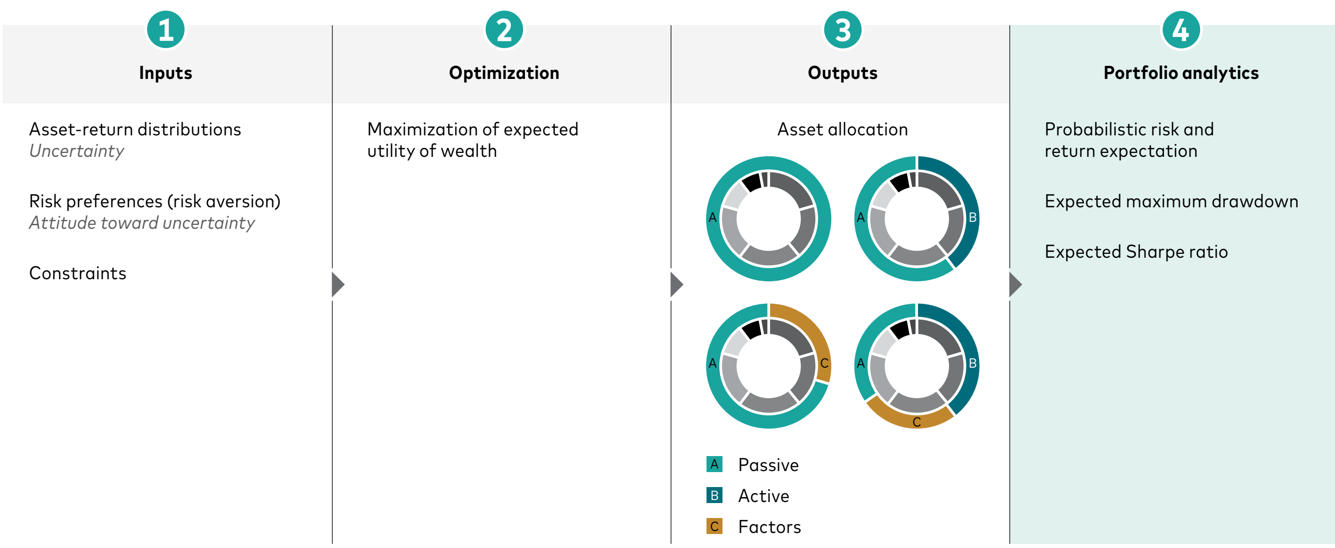
## The Vanguard Asset Allocation Model

The VAAM uses a universal utility function to evaluate the risk-return trade-offs of various portfolio combinations within set constraints. A utility function quantifies an investor's risk-return preferences, converting expected returns or wealth levels into a utility score that reflects the investor's risk tolerance. Essentially, it penalizes volatility based on an investor's level of risk aversion. For example, a more risk-averse investor's utility function will apply a greater penalty to volatility, resulting in a more conservative portfolio allocation.

**Figure 5** illustrates how the VAAM optimizes asset allocation by finding the balance of risk and return that best suits an investor's specific risk aversion, as defined by their utility function. The VAAM accomplishes this by combining three sets of inputs:

1. Asset-return distributions, including expected returns, volatility, and correlations for all asset classes, active strategies, and long-only factors;
2. The investor's attitudes toward risk, including systematic risk aversion, alpha risk aversion, and factor risk aversion; and
3. The investor's portfolio constraints or guardrails.

FIGURE 5
### An overview of the VAAM




**Note:** The central circles in the charts show possible allocations to various assets; they are for illustrative purposes only and do not represent any particular investment.
**Source:** Vanguard.

### Generating asset-return distributions for the VAAM

The VAAM leverages the forecasting abilities of the VCMM, a sophisticated financial simulation tool that projects asset-return distributions. The VCMM also forecasts a distribution of asset volatilities and correlations while being sensitive to initial valuations, nonnormal distributions (such as t-distributions, or those with fatter tails), and linkages between asset returns and the broader economy (see Davis et al., 2014).

The VAAM specifically models the performance of active strategies by simulating potential outcomes of factor-adjusted alphas rather than relying on static-point estimates (see Davis et al., 2018). This approach is a distinctive feature of our model and acknowledges the risk of underperformance inherent in active management. The VAAM's simulation engine employs Monte Carlo methods to generate a nonnormal distribution of these alphas, capturing the idiosyncratic risks unique to active management and enhancing decision-making when choosing the appropriate balance between active and passive investments.

### Investors' attitudes toward risk and expected-utility-based optimization

Typically, riskier assets (such as equities) have higher expected returns and higher risk compared with more conservative assets (such as investment-grade fixed income). An investor's aversion to risky asset classes is called "systematic risk aversion" in our model.

Investors may tolerate systematic risk but show an aversion to alpha risk (or "alpha risk aversion"). For example, an institutional investor may favor high equity allocations with limited alpha risk. The VAAM addresses this by incorporating a penalty for alpha risk aversion in its utility scoring whereby the investor's alpha risk aversion is applied to the factor-adjusted alpha simulations.

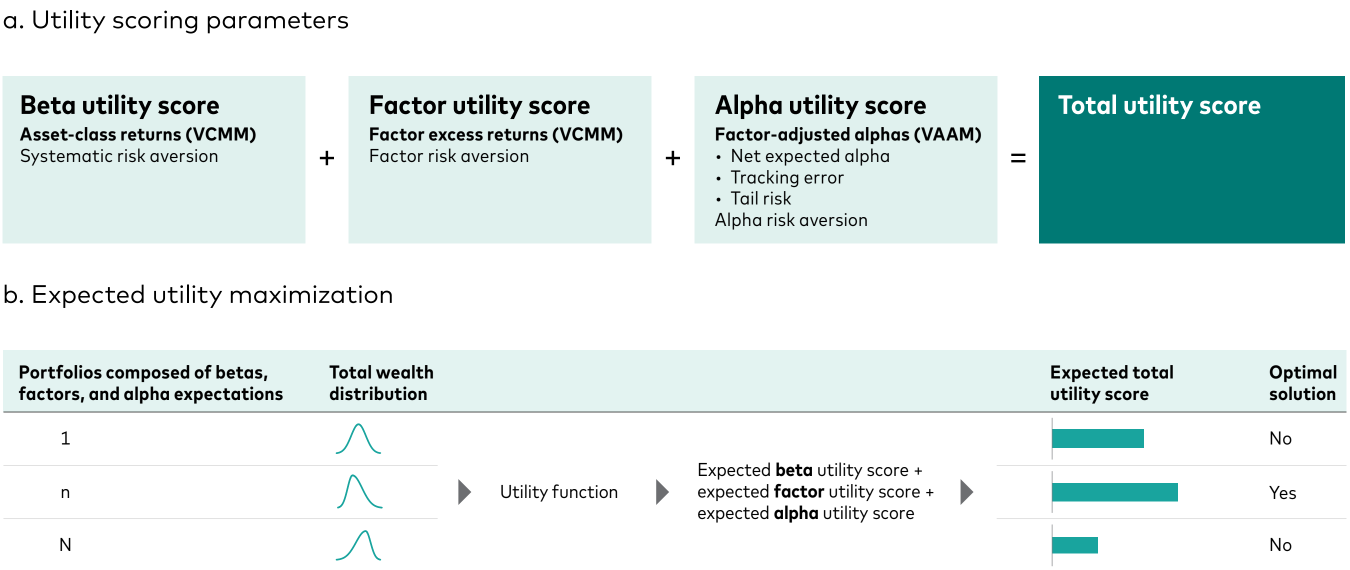
Additionally, the VAAM considers "factor risk aversion," or the aversion to specific factor premia within a portfolio. The model tracks all factor exposures, including those from active managers and direct investments in factor-specific ETFs or funds.

The VAAM optimizes portfolios by maximizing the expected utility of wealth at maturity and penalizing higher levels of risk. We split the utility of wealth into three buckets corresponding to each source of return: active, passive, and factor. The total expected utility score of a portfolio in the VAAM is calculated by adding the individual utility scores for systematic risk, alpha risk, and factor risk. The portfolio with the highest total expected utility score is deemed optimal (see **Figure 6**). The utility scores of each potentially optimal portfolio are averaged across the entire return distribution, considering market and factor simulations along with alpha risk simulations. This allows the VAAM to balance active, passive, and factor allocations while accounting for alpha risk and investor preferences toward different types of risks and uncertainties.[3]

## The VAAM and portfolio constraints

Investors often use portfolio constraints to express their beliefs and avoid "corner solutions," or portfolios that completely exclude certain asset classes. These constraints typically set upper and lower limits on exposures, such as capping REITs at 10% of the total asset allocation, limiting credit exposure to 50% of the total fixed income allocation, or requiring at least 60% of equities to be U.S. stocks. Our model accommodates these linear constraints as well as nonlinear constraints like a target return or income level for a portfolio. Additionally, because the VAAM focuses on long-only investments, it always produces portfolios with positive optimal portfolio weights.

FIGURE 6
### Optimal portfolio selection




**Note:** The information in this figure is for illustrative purposes only and does not represent any particular investment.
**Source:** Vanguard.

3 The VAAM uses an algorithm to efficiently find the optimal portfolio, which can be especially useful when dealing with a large number of assets.

## Expected utility maximization and calibration of risk aversion[4]

Our model aims to maximize the expected utility of an investor's wealth at the end of a set period (10 years, for example) by using a power utility function to reflect the investor's risk preferences. This approach, supported by research from Adler and Kritzman (2007) and Sharpe (2007), suggests that expected utility maximization is more effective than traditional MVO methods. The concept of relative risk aversion has been studied extensively, with research evolving to include experimental and survey methods (see Metrick, 1995, and Barsky et al., 1997).

In our model, risk aversion coefficients do not have any intuitive economic meaning. Moreover, risk aversion coefficients are ordinal, not cardinal, meaning that an investor who is twice as risk-averse as another will have a higher risk aversion coefficient, but not necessarily one that is twice as big.

To address the challenge of calibrating risk aversion, Liu and Xu (2010) recommend an "efficient frontier" approach. This method generates a range of portfolios based on varying initial risk aversion coefficients and selects portfolios that align with an investor's preferences based on risk and performance statistics, such as expected return, volatility, Sharpe ratio, and maximum drawdown. This iterative process continues until the risk and return characteristics of the portfolio match the investor's preferences.

## The mathematics of the VAAM's expected utility maximization

The VAAM uses a power utility function to model an investor's preferences and attitude toward risk:

$$U(W_T) = \begin{cases} \dfrac{W_T^{1-\gamma}}{1-\gamma}, & \gamma > 1 \\ \ln(W_T), & \gamma = 1 \end{cases}$$

Here, $\gamma$ is the relative risk aversion (RRA) coefficient and $W_T$ is the level of terminal wealth relative to starting wealth.

Consider an investor facing the portfolio choice problem presented in investing their wealth over a particular investment horizon. Wealth will compound in each period at the total multiasset portfolio return $R_t$:

$$R_t = \sum_{i=1}^{N} x_i r_{i,t} = \sum_{i=1}^{N} x_i^p r_{i,t}^p + \sum_{i=1}^{N} \sum_{f=1}^{F} x_i^f r_{i,t}^f + \sum_{i=1}^{N} x_i^a r_{i,t}^a$$

$$\begin{cases} r_{i,t}^p = r_{i,t}^M \\ r_{i,t}^f = r_{i,t}^M + \delta_{i,t}^f \\ r_{i,t}^a = \alpha_i + \beta_i r_{i,t}^M + \sum_{f=1}^{F} L_i^f \delta_{i,t}^f + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim t(v)\sqrt{\sigma_{\alpha_i}^2} \end{cases}$$

Here, $x_i$ and $r_i$ are the portfolio weights and relative total returns for each asset class $i$, and superscripts $p$, $f$, and $a$ refer to passive, factor, and active, respectively. The market benchmark return is represented by $r_{i,t}^M$, $\delta_{i,t}^f$ is the excess (to the market benchmark) factor return for factor $f$, and $\beta_i$ and $L_i$ correspond to the market beta and factor loading for each asset class, respectively. $\alpha_i$ is the net (that is, factor-adjusted) excess active return.

The portfolio choice problem consists of finding optimal weights for each passive asset class, factor, or active manager or strategy in the portfolio. We then express the utility-based optimization problem that we want to solve for as:

$$\max_x \mathbb{E}\left[U\left(\frac{W_T}{W_0}\right)\right] \longrightarrow \max_x \left\{\mathbb{E}\left[\frac{W_p^{1-\gamma_p}}{1-\gamma_p}\right] + \mathbb{E}\left[\frac{W_f^{1-\gamma_f}}{1-\gamma_f}\right] + \mathbb{E}\left[\frac{W_a^{1-\gamma_a}}{1-\gamma_a}\right]\right\}$$

$$s.t.\ \{x_i \in \mathbb{R} \mid 0 \le x_i \le 1\} \wedge \sum_i x_i = 1$$

$$\sum_i C \cdot x_i \le b$$

Here, $W_p$, $W_f$, and $W_a$ are the wealth at maturity coming from systematic, factor, and factor-adjusted alpha exposures, respectively; $\gamma_p$, $\gamma_f$, and $\gamma_a$ are the systematic, factor, and alpha risk aversions, respectively; and $C$ and $b$ refer to the set of linear inequality constraints.

4 For more details on this methodology, see Aliaga-Díaz et al. (2020).

## How does the VAAM respond to custom investor inputs?

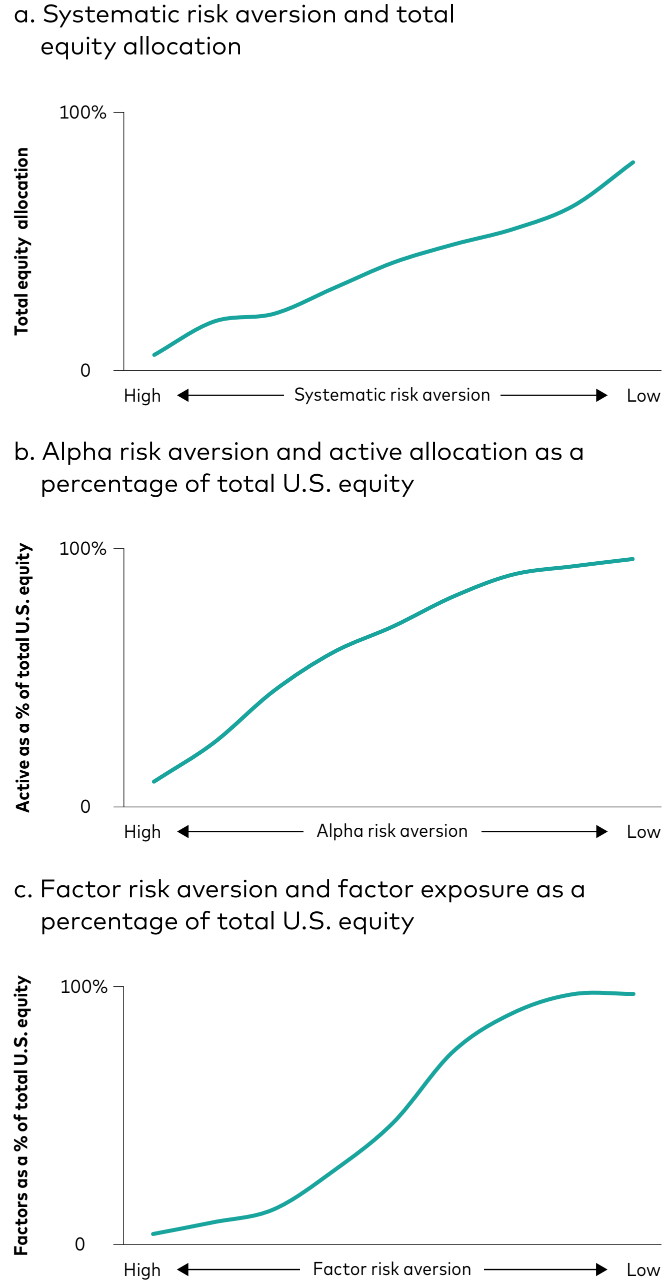
The driving force behind the VAAM has been the aspiration to deliver customized portfolios to investors based on their risk preferences. This requires simultaneously managing multiple trade-offs investors face when building their portfolio amid uncertainty.

To demonstrate the link between risk tolerance and asset allocation, we provide the model with several risk aversion coefficients for systematic, alpha, and factor risk and solve for an optimal portfolio. For instance, all else being equal, one would expect reducing systematic risk aversion—that is, decreasing the penalty toward dispersion of asset-class returns—to increase the equity allocation in a multiasset portfolio. We show this relationship in **Figure 7a**, where the total equity allocation increases as the systematic risk aversion decreases.

By design, lowering alpha risk aversion (the penalty toward dispersion of factor-adjusted alpha distribution) should increase the active allocation in the portfolio. **Figure 7b** confirms that idea, as the allocation to a U.S. active fund as a percentage of the total U.S. equity allocation (active and passive) increases as the alpha risk aversion is lowered.[5] Similarly, **Figure 7c** shows that lowering factor risk aversion increases the allocation to factor exposure in the portfolio as a percentage of U.S. equity allocation. In this example, we assess the allocation of U.S. equity passive factor vehicles relative to total U.S. equity allocation (active, passive, and factor) by keeping the systematic and alpha risk aversion constant.

FIGURE 7

**Asset allocation and risk tolerance levels**




**Notes:** Portfolios have been optimized over a 10-year investment horizon using U.S. equities, global ex-U.S. equities, U.S. bonds, global ex-U.S. bonds, U.S. intermediate-term credit bonds, and U.S. short-term credit bonds. Global ex-U.S. bonds are hedged to U.S. dollars. The following constraints apply: global ex-U.S. equities, up to 40% of the total equity allocation; global ex-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); U.S. intermediate-term credit bonds, up to 60% of total credit bonds; and U.S. short-term credit bonds, up to 60% of total credit bonds. Active and factor allocation options are considered for U.S. equities only. Market beta, factor loadings, expected alpha, and tracking error estimates for the U.S. equities active fund are as shown in Figure 3, on page 5.

**Sources:** Vanguard calculations, using asset-return projections from the March 31, 2024, running of the VCMM.

5 The active fund used in this example is the same as the one reported in Figure 3, on page 5. In all the examples in Figure 7, we consider the option to include exposure toward alpha risk and factor risk in the portfolio for U.S. equities only.

What is the impact of lowering factor-adjusted alpha expectations? **Figure 8** shows four portfolios, two of which (Portfolios A and B) have identical inputs except for the gross factor-adjusted expected alpha, which is 83 bps for Portfolio A and 11 bps for Portfolio B. Lowering the factor-adjusted expected alpha reduces the allocation to the U.S. active fund from 60% to 21%.

We also show the impact of lowering dispersion around the factor-adjusted alpha. Portfolios C and D show that in this case the allocation toward active assets increases. In this example, the dispersion parameter (or tracking error) of the active fund reported in Figure 3 was lowered from 4.03% for Portfolio C to 2.00% for Portfolio D, and the alpha risk aversion was increased for both in order to compare portfolios with significant difference in the active allocation. When evaluating these results, it is important to recognize the significant increase in the active manager's skill as measured by the information ratio. By virtue of the tracking error being reduced, the manager's excess return per unit of risk (or information ratio) has increased from 0.21 (83 bps/403 bps) to 0.41 (83 bps/200 bps).

FIGURE 8

### Optimized asset allocations

| Category | Component | U.S. equity allocation | Portfolio A | Portfolio B | Portfolio C | Portfolio D |
|---|---|---|---|---|---|---|
| Risk aversion | Systematic | | Medium | Medium | Medium | Medium |
| | Alpha | | Low | Low | Medium | Medium |
| | Factor | | Medium | Medium | Medium | Medium |
| Active risk characteristics | Factor-adjusted expected alpha | | 83 bps | 11 bps | 83 bps | 83 bps |
| | Tracking error | | 4.03% | 4.03% | 4.03% | 2.00% |
| Asset class weights | U.S. equities | | 60% | 29% | 26% | 41% |
| | | Active | 60% | 21% | 18% | 36% |
| | | Value factor | 0% | 6% | 6% | 4% |
| | | Illiquidity factor | 0% | 2% | 2% | 1% |
| | Global ex-U.S. equities | | 24% | 19% | 18% | 27% |
| | U.S. bonds | | 14% | 39% | 27% | 17% |
| | Global ex-U.S. bonds | | 2% | 1% | 2% | 0% |
| | U.S. intermediate-term credit bonds | | 0% | 6% | 16% | 9% |
| | U.S. short-term credit bonds | | 0% | 6% | 11% | 6% |
| | Total | | 100% | 100% | 100% | 100% |
| Summary statistics | Total equity allocation | | 84% | 48% | 44% | 68% |
| | Expected return | | 6.1% | 5.9% | 6.0% | 6.3% |
| | Expected volatility | | 13.8% | 8.1% | 7.6% | 11.1% |

**Notes:** Portfolios have been optimized over a 10-year investment horizon using U.S. equities, global ex-U.S. equities, U.S. bonds, global ex-U.S. bonds, U.S. intermediate-term credit bonds, and U.S. short-term credit bonds. Global ex-U.S. bonds are hedged to U.S. dollars. The following constraints apply: global ex-U.S. equities, up to 40% of the total equity allocation; global ex-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); U.S. intermediate-term credit bonds, up to 60% of total credit bonds; and U.S. short-term credit bonds, up to 60% of total credit bonds. Market beta, factor loadings, and tracking error estimates for the U.S. equities active fund are as reported in Figure 3, on page 5. Factor-adjusted expected alpha data are assumed to be before fees.

**Sources:** Vanguard calculations, using asset-return projections from the March 31, 2024, running of the VCMM.

## Role of the VAAM in the portfolio construction process

### General application

As discussed in Aliaga-Díaz et al. (2022), there are several dimensions to consider when constructing a portfolio around an investor's financial goals: the type of goals, the assets to be considered, asset-return expectations, and the investor's investment horizon and risk tolerance.

Model-free approaches may yield reasonable portfolios when no more than two or three asset classes are considered. However, for more complex portfolios, with numerous asset classes and sub-asset classes, active funds, and time-varying returns, investment methodologies that explicitly incorporate assumptions around risk and return and an investor's attitude toward them may offer a better option compared with simpler, model-free portfolios. To that end, the VAAM effectively accounts for multiple dimensions in crafting portfolio solutions for investors who are willing to embrace model forecast risk.

In terms of investor goals,[6] there are three common investment objectives that can be solved using the VAAM:

1. **Wealth growth:** Achieving maximum wealth growth over the long term within the limits of a risk profile that is acceptable to the investor;
2. **Risk hedging:** Mitigating or controlling for specific investment risks, such as inflation risk, duration risk (sensitivity to changes in interest rates), and projected portfolio volatility targets; and
3. **Return target:** Seeking a certain level of portfolio payout or return over time.

These objectives, combined with various investor preferences and beliefs and their corresponding investment methodologies, lead to 13 potential portfolio solutions. The VAAM's flexibility in incorporating an investor's preferences and risk tolerance extends its utility beyond merely addressing active-passive investment decisions. It also facilitates solutions like strategic factor model portfolios, return-target portfolios, and customized asset allocations for diverse goals, as shown in **Figure 9**.

FIGURE 9

**Portfolio solutions map**

| Goals | Investment methodologies | Portfolio solutions | | | |
|---|---|---|---|---|---|
| Wealth growth | Market cap-weighted | Market cap portfolios | | | |
| | Model-based strategic asset allocation (SAA) | Factor premia portfolios | | | |
| | Active-passive methodology | Traditional active-passive portfolios | Private asset portfolios | ESG portfolios | Direct-indexing portfolios |
| | Time-varying asset allocation (TVAA) | Time-varying portfolios | Tax-efficient portfolios | | |
| Risk hedging | Model-based SAA | Inflation-hedging portfolios | Duration-target portfolios | | |
| | TVAA | Risk-target portfolios* | | | |
| Return target | TVAA | Total return-target portfolios | Income-target portfolios | | |

* Vanguard is actively conducting research on the methodologies for risk-target portfolios.

**Source:** Aliaga-Díaz et al., 2022.

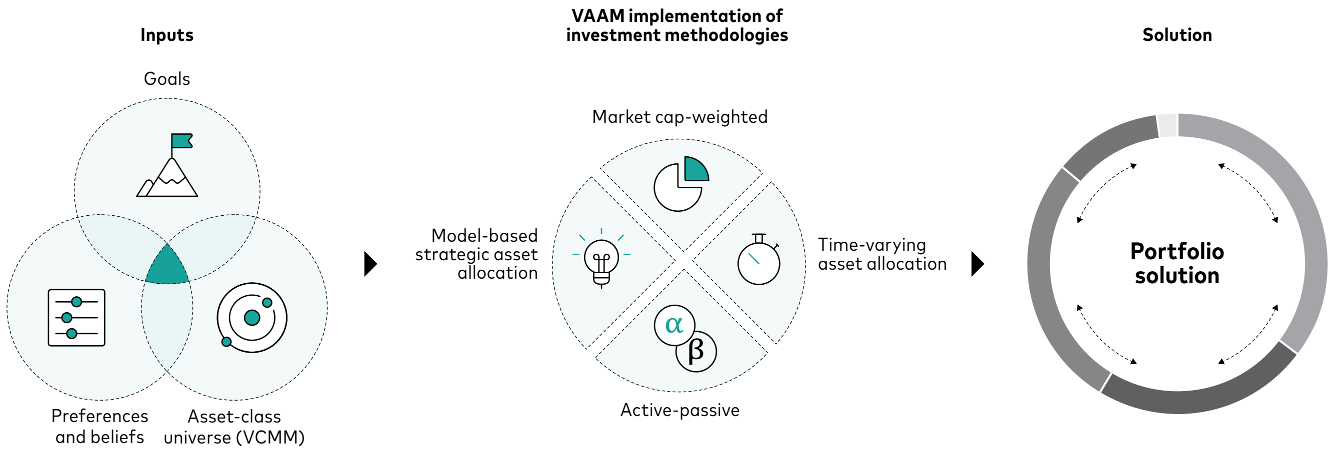
6 Investor goals can be grouped into two broad categories: life-cycle personal goals and financial goals. Financial goals require a different type of portfolio than goals-based glide paths, since financial goals are investment solutions that are independent of any calendar date or the investor's age. In this paper, we cover portfolio solutions that are designed to achieve financial goals.

Once an investor's goals, investment horizon, preferences, and risk tolerance are established, the set of eligible investments can be defined. Distributions of asset returns from the VCMM, which include volatility and correlations in addition to return expectations, are used as an input for each eligible asset. The VAAM then balances the potential benefits and risks of each investment, guided by the investor's risk tolerance, via the utility-driven optimization.

Portfolio customizations can also include investors implementing portfolio constraints to align with their beliefs. Based on investor preferences, the VAAM can also incorporate various investment methodologies such as strategic asset allocation, active-passive, and time-varying asset allocation to create a portfolio solution, as shown in **Figure 10**.[7]

**FIGURE 10**

**Portfolio solutions depend on an investor's goals and preferences**




**Source:** Aliaga-Díaz et al., 2022.

**7** The VAAM restricts portfolios to long-only investments, ensuring all asset weights are positive.

Below we describe in detail three portfolio solutions: strategic model portfolios, active-passive portfolios, and return-target portfolios.

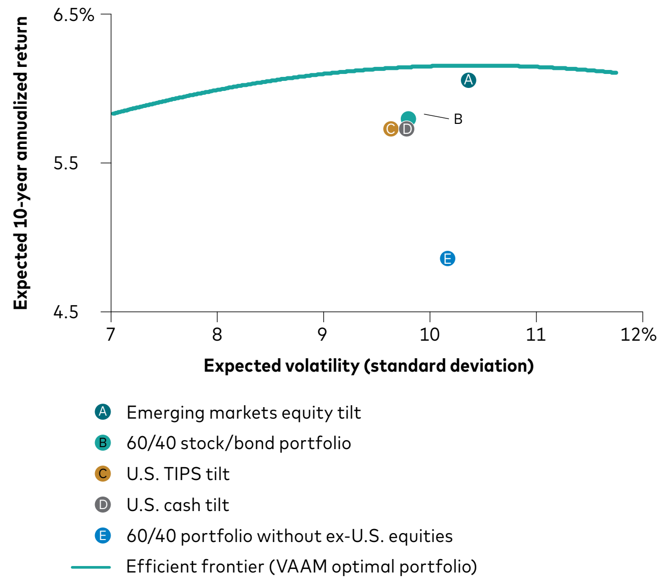
### 1. Strategic model portfolios

For an investor looking to get exposure to the markets through passive funds or ETFs, the VAAM can be used to deliver a traditional, passive-only asset allocation. The investor in this case has two levers: They can specify their opportunity set (the passive investments and asset classes that should be considered in the portfolio) and their risk preferences (their preferred level of systematic risk aversion and any relevant constraints).

The efficient frontier shown in **Figure 11** was constructed following this approach. The dots in the figure represent portfolios with popular ad hoc tilts, in contrast to the VAAM optimization approach. In an attempt to increase portfolio returns, for instance, investors might decide to overweight riskier assets, such as emerging markets equities. Home bias could also cause some investors to shy away from globally diversified portfolios. The problem with such ad hoc tilts is that they ignore correlations among assets and can lead to inefficient portfolios. Portfolio tilts may be better assessed within an optimization framework. The efficient frontier in Figure 11, which was constructed using the same traditional asset classes used for the individual tilts, still lies above the ad hoc portfolios in terms of expected return, illustrating the value added by an optimization approach. (For more information about the construction of an efficient frontier, see page 9.)

FIGURE 11

**Portfolio tilts should be assessed within an optimization framework**




**Notes:** For the efficient frontier, portfolios have been optimized over a 10-year investment horizon using U.S. equities, global ex-U.S. equities, U.S. bonds, global ex-U.S. bonds, U.S. intermediate-term credit bonds, and U.S. short-term credit bonds. Global ex-U.S. bonds are hedged to U.S. dollars. The following constraints apply: global ex-U.S. equities, up to 40% of the total equity allocation; global ex-U.S. bonds, up to 50% of total (noncredit) bonds; total credit bonds, up to 50% of total fixed income (bonds and credit bonds); U.S. intermediate-term credit bonds, up to 60% of total credit bonds; and U.S. short-term credit bonds, up to 60% of total credit bonds. The 60/40 stock/bond portfolio has the following asset allocations: 36% U.S. equities, 24% global ex-U.S. equities, 28% U.S. bonds, and 12% global ex-U.S. bonds. Portfolios with tilts include a 10% tilt from the 60/40 stock/bond portfolio to the asset specified, with the fixed income tilts funded from the fixed income allocation and the equity tilts funded from the equity allocation.

**Sources:** Vanguard calculations, using asset-return projections from the March 31, 2024, running of the VCMM.

## 2. Active-passive portfolios

The VAAM can also help investors harvest an active manager's alpha via active-passive portfolio solutions. For many investors, active investing amounts to blending active investments with passive ones. The decision to take active risk is just another form of risk-return trade-off in investing; active investments offer the potential to outperform a given benchmark, but they also introduce the risk of underperformance.

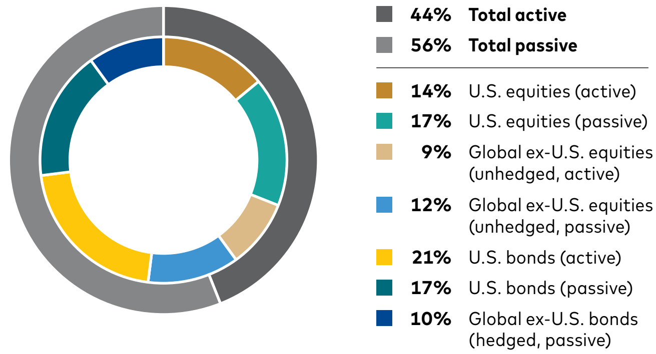
To address this risk, Vanguard's Active-Passive Decision Framework (see Wallick et al., 2017, and Aliaga-Díaz et al., 2024) quantifies expectations for both estimated outperformance (expected alpha) and active risk (tracking error and the odds of underperforming a passive benchmark), then weighs them against each other to tailor an active-passive mix based on the investor's risk tolerance. This methodology is built into the VAAM.

**Figure 12** shows an active-passive portfolio and compares it against a 60% stock/40% bond market cap-weighted benchmark. The active-passive portfolio considered in this example (including active U.S. equity assets, active U.S. core bond assets, and global ex-U.S. active equity assets) produces a median excess return of 1.3 percentage points relative to a 60/40 portfolio (7.2% versus 5.9%) with a higher median risk-adjusted return, as measured by the Sharpe ratio (0.25 versus 0.14). This comes at the expense of a tracking error of 0.6% and translates into a 18.8% probability of underperforming the benchmark in any given year.

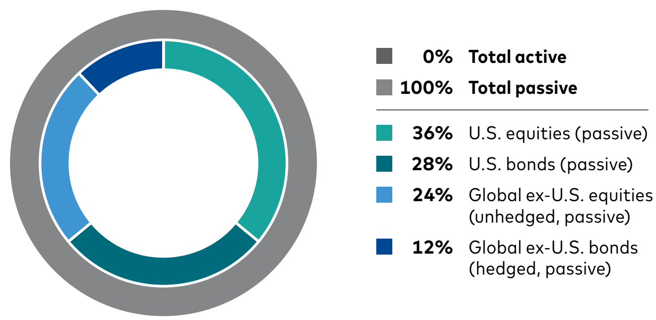
FIGURE 12

### Active-passive portfolios and their risk-return trade-offs

a. Active-passive portfolio




b. 60/40 market cap-weighted benchmark




c. Analytics

| | Active-passive | 60/40 |
|---|---|---|
| Annualized total return | 7.2% | 5.9% |
| Annualized volatility | 8.5% | 9.7% |
| Excess return | 1.3% | — |
| Probability of underperformance | 18.8% | — |
| Tracking error | 0.6% | — |
| Maximum drawdown | –30.1% | –39.3% |
| Sharpe ratio | 0.25 | 0.14 |

**Notes:** Active-passive portfolio allocations were determined by the VAAM. The active funds assumed are hypothetical and do not reflect a specific fund as the portfolios are for illustrative purposes. The assets under consideration were indexed and active U.S. and global ex-U.S. equities and fixed income. The alpha and tracking error for the active funds were 0.45% alpha and 1.5% tracking error for U.S. equities, 0.34% alpha and 0.69% tracking error for global ex-U.S. equities, and 0.84% alpha and 2.8% tracking error for U.S. bonds. Maximum drawdown is the median peak-to-trough drop in the portfolio's value in 10,000 VCMM simulations for any given year. Asset-return projections are from the March 31, 2024, running of the VCMM.

**Source:** Vanguard.

### 3. Return-target portfolios

Return-target portfolios may be appropriate for an investor who needs to target a particular level of return or portfolio payout in order to fund a required level of spending from the portfolio. This is a common goal for institutional investors such as endowments and foundations, but it can also be relevant for individual investors, model portfolios, and multiasset funds. Achieving a desired target payout through changing market conditions may require adjusting the asset allocation over time.

**Figure 13**, on page 17, shows return-target portfolio allocations corresponding to a 4% expected return target over a 10-year horizon. By design using a time-varying asset allocation (TVAA) methodology, the expected return of these portfolios remains at or above 4%. In contrast, the market cap-weighted portfolio (a static 60% stock/40% bond benchmark) falls short of this return expectation in December 2017, when the annualized return is 3.8%, and again in December 2021, when it's 3.9%.

The return-target portfolio approach identifies a portfolio that best serves the return objective given an investor's risk tolerance. While the expected outcomes of achieving the desired level of return are better for the return-target portfolio than the static benchmark, it is also possible for a return-target portfolio to underperform in comparison, especially over shorter periods. In fact, one downside of time-varying portfolios is the higher portfolio risk (in terms of both volatility and maximum drawdown) that may be needed at times to attain the targeted return goal, especially in a low-return environment.

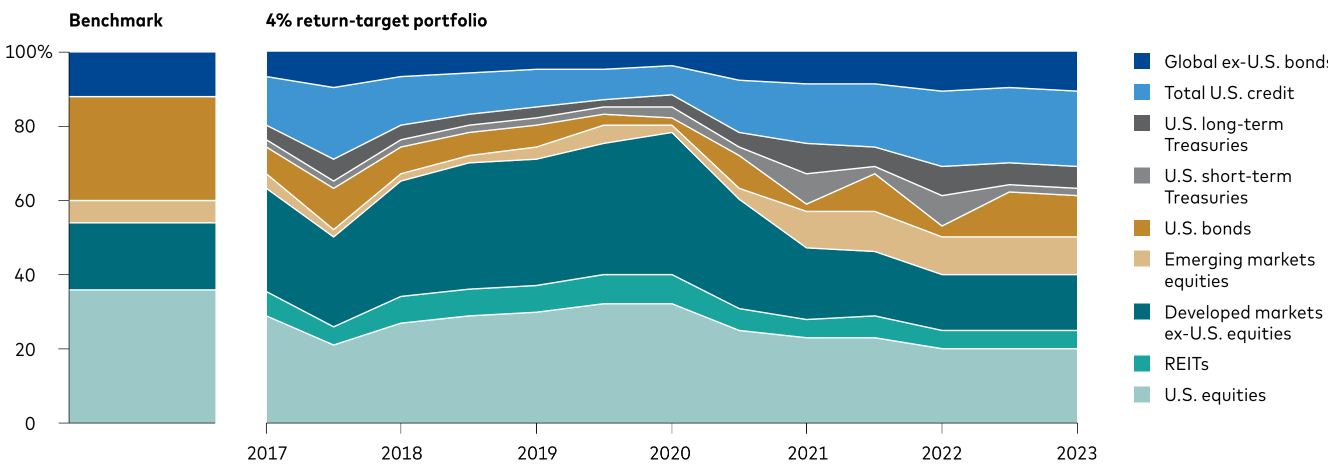
FIGURE 13

## Return-target portfolios adjust to market conditions

a. VAAM optimal portfolio allocations through time

| 10-year median expected portfolio analytics | | Equity allocation | Annualized total return | Annualized volatility | Probability of meeting the 4% return target | Probability of loss of 10% or more in any given year | Maximum drawdown | Sharpe ratio |
|---|---|---|---|---|---|---|---|---|
| **December 2017** | 4% RTP | 67% | 4.0% | 9.0% | 50.2% | 23.8% | −41.6% | 0.21 |
| | Benchmark | 60% | 3.8% | 9.2% | 43.5% | 20.7% | −37.9% | 0.19 |
| **December 2018** | 4% RTP | 67% | 6.2% | 10.0% | 85.7% | 40.3% | −33.3% | 0.34 |
| | Benchmark | 60% | 5.5% | 9.4% | 77.5% | 39.0% | −39.1% | 0.29 |
| **December 2019** | 4% RTP | 74% | 5.4% | 11.0% | 72.7% | 66.3% | −42.7% | 0.29 |
| | Benchmark | 60% | 4.7% | 9.4% | 63.5% | 49.4% | −39.8% | 0.27 |
| **December 2020** | 4% RTP | 80% | 5.4% | 11.9% | 71.0% | 84.1% | −50.9% | 0.32 |
| | Benchmark | 60% | 4.2% | 9.4% | 53.5% | 54.5% | −46.5% | 0.29 |
| **December 2021** | 4% RTP | 57% | 4.1% | 9.0% | 52.0% | 47.8% | −40.0% | 0.24 |
| | Benchmark | 60% | 3.9% | 9.3% | 47.4% | 56.4% | −44.7% | 0.21 |
| **December 2022** | 4% RTP | 50% | 6.2% | 8.4% | 89.5% | 20.7% | −31.7% | 0.27 |
| | Benchmark | 60% | 6.1% | 9.7% | 83.3% | 37.9% | −40.9% | 0.22 |
| **December 2023** | 4% RTP | 45% | 6.1% | 8.5% | 88.4% | 21.9% | −30.7% | 0.25 |
| | Benchmark | 60% | 5.9% | 9.7% | 81.2% | 35.5% | −39.3% | 0.14 |

b. Return-target portfolio allocations through time compared with benchmark




**Notes:** The 10-year expected portfolio data are calculated based on 10,000 VCMM 10-year simulations from December 2017 through December 2023. The benchmark is a market cap-weighted 60/40 stock/bond portfolio with the following asset allocations: 36% U.S. equities, 18% developed markets ex-U.S. equities, 6% emerging markets equities, 28% U.S. bonds, and 12% global ex-U.S. bonds. 4% RTP represents a 4% return-target portfolio, which is constructed using TVAA methodology. Maximum drawdown is the median peak-to-trough drop in the portfolio's value in 10,000 VCMM simulations for any given year.

**Sources:** Vanguard calculations, using asset-return projections from the March 31, 2024, running of the VCMM.

## The VAAM's caveats and limitations

Like any model, the VAAM has its assumptions, limitations, and measurement imprecisions. It relies on VCMM forecasts, which are based on historical data but also include forward-looking return assumptions and assumptions about asset-class risk characteristics. The accuracy of these VCMM forecasts, and thus the optimal weights the VAAM calculates, can be compromised if historical data do not accurately predict future conditions. Additionally, the factor decomposition used to estimate factor-adjusted alpha and tracking error (as shown in Figures 2 and 3) is prone to estimation errors, leading to parameter uncertainty.

The VAAM offers a simplified representation of reality and may not capture the complex decision-making processes of every investor. It assumes a universal power utility function for all investors, which, despite its common acceptance, might not accurately reflect every attitude toward risk and expected wealth. Furthermore, the VAAM uses an algorithm to determine optimal portfolio weights, a stochastic method that introduces randomness and potential variability in the results. Despite these limitations, the VAAM provides a valuable framework for addressing multidimensional asset allocation challenges involving systematic, alpha, and factor risks.

## Conclusion

The VAAM is a proprietary asset allocation model that integrates active, passive, and factor-based strategies, tailored to the uncertainties in active returns and investors' risk preferences. It leverages the VCMM forecasting framework, which accounts for factors like initial valuations, forward-looking assumptions, nonnormal distributions, and the interplay between asset returns and macroeconomic factors.

The VAAM has multiple research and business applications. It enhances advisory services by integrating seamlessly with digital platforms, allowing for customized portfolio solutions that maintain scalability and consistency in investment approaches. From a regulatory standpoint, the VAAM's quantitative nature enhances transparency in the asset allocation process, simplifying oversight and review on advisory platforms and within investment committees.

Behaviorally, the VAAM clarifies the asset allocation process by turning implicit decisions, such as setting alpha expectations and understanding risk aversion, into explicit ones. This transparency helps in scrutinizing assumptions typically made subconsciously in traditional asset allocation, promoting a more informed investment decision-making process.

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## About the Vanguard Capital Markets Model

**IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. VCMM results will vary with each use and over time.**

The VCMM projections are based on a statistical analysis of historical data. Future returns may behave differently from the historical patterns captured in the VCMM. More importantly, the VCMM may be underestimating extreme negative scenarios unobserved in the historical period on which the model estimation is based.

The VCMM is a proprietary financial simulation tool developed and maintained by Vanguard's primary investment research and advice teams. The model forecasts distributions of future returns for a wide array of broad asset classes. Those asset classes include U.S. and international equity markets, several maturities of the U.S. Treasury and corporate fixed income markets, international fixed income markets, U.S. money markets, commodities, and certain alternative investment strategies. The theoretical and empirical foundation for the VCMM is that the returns of various asset classes reflect the compensation investors require for bearing different types of systematic risk (beta). At the core of the model are estimates of the dynamic statistical relationship between risk factors and asset returns, obtained from statistical analysis based on available monthly financial and economic data from as early as 1960. Using a system of estimated equations, the model then applies a Monte Carlo simulation method to project the estimated interrelationships among risk factors and asset classes as well as uncertainty and randomness over time. The model generates a large set of simulated outcomes for each asset class over several time horizons. Forecasts are obtained by computing measures of central tendency in these simulations. Results produced by the tool will vary with each use and over time.

## Indexes for VCMM simulations

The long-term returns of our hypothetical portfolios are based on data for the appropriate market indexes as of March 31, 2024. We chose these benchmarks to provide the most complete history possible, and we apportioned the global allocations to align with Vanguard's guidance in constructing diversified portfolios.

Asset classes and their representative forecast indexes are as follows:

- U.S. equities: MSCI US Broad Market Index.
- Global ex-U.S. equities: MSCI All Country World ex USA Index.
- Developed markets ex-U.S. equities: MSCI World ex-U.S. Equity Index.
- Emerging markets equities: MSCI Emerging Market Index.
- U.S. REITs: FTSE/NAREIT US Real Estate Index.
- U.S. bonds: Bloomberg U.S. Aggregate Bond Index.
- Global ex-U.S. bonds (hedged): Bloomberg Global Aggregate ex-USD Index.
- U.S. Treasury bonds: Bloomberg U.S. Treasury Index.
- U.S. short-term Treasuries: Bloomberg U.S. 1–5 Year Treasury Bond Index.
- U.S. intermediate-term Treasuries: Bloomberg U.S. 5–10 Year Treasury Bond Index.
- U.S. long-term Treasuries: Bloomberg U.S. Long Treasury Bond Index.
- U.S. short-term credit: Bloomberg U.S. 1–5 Year Credit Index.
- U.S. intermediate-term credit: Bloomberg U.S. 5–10 Year Credit Index.
- U.S. long-term credit: Bloomberg U.S. Long Credit Index.
- U.S. high-yield corporate: Bloomberg U.S. High Yield Corporate Bond Index.
- Emerging markets bonds: Bloomberg Emerging Markets USD Sovereign Bond Index – 10% Country Capped.
- U.S. TIPS: Bloomberg U.S. Treasury Inflation Protected Securities Index.
- U.S. cash: U.S. 3-Month Treasury—constant maturity.
- U.S. mortgage-backed securities: Bloomberg U.S. Mortgage Backed Securities Index.

See Appendix A on page 22 for more information about the U.S. equity style factors used for the analysis in this paper.

## Appendix A. U.S. equity style factor definitions

This table shows the criteria that were used to define and construct U.S. equity style factors for the analysis reported in this paper. The VAAM's methodology is not dependent on the definition of factors shown below, and the model can accommodate any other factor definition or benchmark.

| Factor | Data starting point | Succinct definition | Selection universe | Weighting scheme |
|---|---|---|---|---|
| Value | January 1980 | 1/3 of stocks with the lowest price-to-book ratio | Russell 1000 | Market capitalization-weighted |
| Growth | January 1980 | 1/3 of stocks with the highest price-to-book ratio | Russell 1000 | Market capitalization-weighted |
| Large-cap | January 1980 | 2/3 of stocks with the highest market capitalization | Russell 1000 | Market capitalization-weighted |
| Mid-cap | January 1980 | 1/3 of stocks with the lowest market capitalization | Russell 1000 | Market capitalization-weighted |
| Small-cap | January 1980 | 2/3 of stocks with the lowest market capitalization | Russell 3000 | Market capitalization-weighted |
| Momentum | January 1980 | 1/3 of stocks with the highest 12-month trailing returns | Russell 1000 | Market capitalization-weighted |
| Low volatility | January 1980 | 1/5 of stocks with the lowest annualized return volatility | Russell 1000 | The inverse of volatility |
| Quality | January 1987 | 1/3 of stocks with the highest profitability and lowest investment | Russell 1000 | Market capitalization multiplied by the quality score (the quality score is defined as half of the sum of the profitability score and the investment score) |
| Liquidity | January 1992 | 1/3 of stocks with the lowest liquidity score | Russell 3000 | Market capitalization multiplied by the illiquidity score (the illiquidity score is defined as 1/3 of the sum of share turnover, dollar turnover, and Amihud illiquidity) |

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